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ANGULAR ESTIMATION[1]

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In one study, 10 Ss estimated the directional trend (heading) of simulated radar trails, using different response modes; rotary switch adjustment permitted better accuracy than numerical estimation. Varying the displayed length of the simulated trails from 5/16 to 1½ inches had no apparent effect on estimation accuracy. 5 civilian Ss proved more accurate than 5 airmen. In a 2nd study, 20 Ss estimated the angular position of lines varying in length from ⅛ to 1 inch, using equipment which permitted switch adjustment and numerical estimation only to the nearest 10 degrees. Results were the same as before. In addition, this report notes differences in estimation accuracy and bias related to the actual angle of displayed lines over a 360-degree range, as well as biasing effects of right- vs. left-handed switch adjustment.

In man-machine systems one of the functions frequently allotted to man is the estimation of directional or angular relationships. Human ability to make such estimates is the basis of our motor skills—for a relatively simple response sequence such as picking up a pencil just as for considerably more complex activities such as piloting an aircraft, perception of directional relations is continually required of us. This paper reports the results of two experimental studies exploring human ability to estimate angular relations. In one case, subjects were asked to report the directional trend ("heading") of a series of radar returns on a simulated display. In the other, they judged the angular position of displayed vectors, or straight lines.

For a human operator performing an air surveillance function, viewing raw radar returns on a PPI scope or filtered data on some sort of situation display, equipment facilities are generally adequate and the perception of simple two-dimensional directional relations is comparatively easy. Whether he must note a particular position, or must judge the bearing of one point with respect to another, or must estimate the heading (projected direction) of a series of returns, the stimulus information he requires is available on the display. In such a situation, however, difficulties may arise when the operator must respond, or report his observations. The means provided him to report directional relations may influence the ease and accuracy with which he can do this job.

In the case of positional designation, our common experience assures us that pointing, when possible, is preferable to verbal description. Happily, this compelling conviction is confirmed by published studies, as for example one by Reed and Bartlett (1947). As for estimation of direction, our experience suggests that here also the equivalent of pointing would be preferable to verbalization. The two studies described in this report confirm this view.

Experiment I

Procedure

Ten subjects were asked to make heading estimates for a series of 60 simulated radar trails. Each man went through this series four times (with the simulated trails presented in a different random order each time) using four different response modes for indicating his estimate. The subjects were instructed to make their estimates as accurately and quickly as possible. Five of the subjects were airmen, with operational experience in the use of an eight-position detented rotary switch for reporting heading estimates. The other five subjects were civilian colleagues at MITRE with no specific experience in making such heading estimates.

The simulated radar trails consisted of strings of black dots inked on white paper. The latest "return," which had to be distinctive to permit heading estimation, was indicated by a larger dot. The simulated trails were displayed in three different lengths to determine the extent to which this factor influences heading estimation. The lengths used were 5/16 inches, 1 inch, and 1½ inches as measured on the display sur-

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face, with 20 trails displayed of each length. The choice of lengths was related to a particular design application in the Air Force SAGE (Semi-Automatic Ground Environment) System and has no theoretical basis. Subjects' viewing distance to the display surface was approximately 18 inches.

Each subject estimated headings for the series of 60 simulated trails four times, using four different response modes, a total of 240 estimates. Two of the response modes involved adjustment of a rotary switch, mounted in one case to the right of the displayed tracks for right-handed use (all subjects were right-handed) and in the other case to the left for left-handed adjustment. This switch had a black, circular knob, 2¼ inches in diameter and 1 inch thick, with a white arrow painted across its diameter. This knob was continuously adjustable, and no reference lines were provided on its mounting base. The subject was simply asked to turn the knob so that the arrow pointed in the same direction as the displayed radar trail.

In the other two response modes, the subjects were asked to give numerical heading estimates from 1 to 360, to the nearest degree if they could. The orientation used was that a trail heading directly up would be called "360," to the right was "90," directly down was "180," etc. In one response condition, the subjects made these estimates with no external reference. In the other, subjects were encouraged to refer to a sample azimuth circle (8-inch diameter with 1-degree intervals and 10-degree labeling) placed on the table before them.

Each subject used the response modes in a different sequence to balance possible order effects. The experimenter recorded the error for each estimate made, and the total time required. The subjects were given no indication of the accuracy of their estimates at any time during the experiment.

TABLE 1

HEADING ESTIMATION ERRORS (in degrees) FOR FIRST EXPERIMENT

Subjects	Length of displayed radar trail		
	$\frac{5}{16}''$	$1''$	$1\frac{1}{2}''$
Airmen			
Switch adjustment			
Right-handed	6.6	7.6	6.7
Left-handed	7.6	6.6	9.6
Numerical estimation			
Unaided	11.1	9.4	11.2
Aided	10.4	8.3	11.8
Civilian			
Switch adjustment			
Right-handed	4.6	4.7	4.8
Left-handed	4.0	4.4	4.1
Numerical estimation			
Unaided	7.2	6.9	6.8
Aided	4.5	6.1	5.6

Note.—Each entry is the mean error of 100 estimates, 20 made by each of 5 subjects.

RESULTS AND DISCUSSION

There were no consistent differences in estimation accuracy for tracks of different displayed length. As a group, the civilian subjects were more accurate in estimating the displayed headings than were the airmen. Both subject groups were more accurate using rotary switch adjustment than when they were required to make numerical estimates of heading. A summary of average estimation error is presented in Table 1.

Analysis of variance of estimation errors was undertaken along the lines recommended by Edwards (1950) for data involving repeated measurements from the same subjects. This variance analysis was based on the sums of each subject's 20 estimation errors under each combination of response mode and display length. Subject groups differed significantly at $p < .001$ ($F = 19.5$, $df = 1/8$). Response modes were significantly different at the same level ($F = 14.4$, $df = 3/105$). Display length had no significant effect, nor did any interaction term.

Duncan's range test (1955) was applied to the differences among response modes. There was no significant difference between right- and left-handed switch adjustment, nor between aided and unaided numerical estimation. However, both methods of switch adjustment were superior to both modes of numerical estimation at the .01 level of significance.

In terms of speed, the average time required per estimate seemed more than anything else to be characteristic of each particular subject, from one individual to another ranging from 3.6 to 7.0 seconds. Because of this variability, no difference between subject groups was demonstrated. An analysis of variance based on speed of response, comparable to that already described, indicated that the only statistically reliable differences were among response modes. The range test confirms at the .01 level that unaided numerical estimation was quicker (4.7 seconds) than either right- or left-handed switch adjustment (5.6 and 5.4 seconds). Aided numerical estimates averaged 5.2 seconds.

The results with regard to estimation error deserve amplification. For certain practical applications something further must be known about the expected distribution of operator heading estimation errors than simply the mean. Figure 1 presents the cumulative frequency curves for errors of increasing size for all subjects. Since the data analysis did not reveal any reliable differences in accuracy between right- and left-handed switch adjustment, or between aided and unaided numerical estimation, all adjustment data are plotted as one curve and numerical estimation data as another. The greater accuracy of switch adjustment is illustrated: 58% of the switch adjustments, for example, were within 5 degrees of the displayed heading, whereas only 48% of numerical estimates were this accurate.

Previous investigators (Chapanis, 1951; Hunt & Warrick, 1957) have noted differences between right- and left-handed adjustment responses. The present data also show such differences, not in terms of average *size* of error but in terms of direction or *bias* of error. There was a tendency for right-handed adjustment to produce relatively fewer clockwise errors (36%) than left-handed adjustment (65%). Chi square analysis of the directional frequency of errors confirmed this difference at the .001 level.

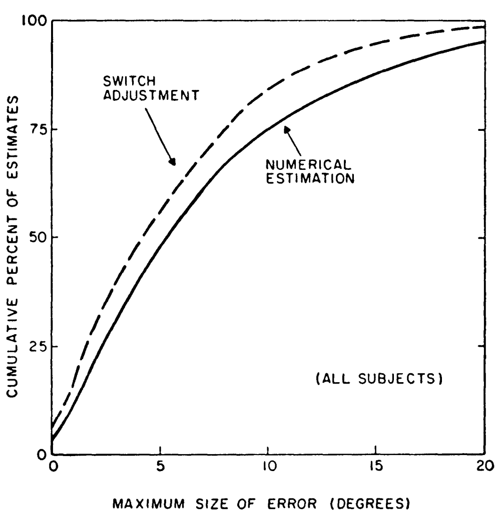

FIG. 1. Cumulative error distribution in first experiment.

Another factor of interest is that, although encouraged to "estimate to the nearest degree if you can," the subjects actually assumed self-imposed response limitations: 90% of their numerical responses were multiples of 5, ending either in 5 or 0. Switch settings were, of course, much more evenly distributed. A similar finding of response quantizing is reported by Chapanis (1951).

The failure to discover any consistent influence of displayed length of the simulated radar trails on heading estimation accuracy is surprising. We might expect that longer trails would provide more adequate stimulus cues to direction, and hence more accurate estimation. It is certainly clear that for extremely short trails heading estimation would deteriorate, and in the limiting case of a single displayed point it would become impossible. The fact that such an effect was not demonstrated over a range of displayed lengths reducing from $1\frac{1}{2}$ to $\frac{5}{16}$ inches suggests that even shorter tracks may constitute an adequate stimulus for heading estimation. This possibility was investigated in the second study described in this report.

EXPERIMENT II

Procedure

The experimental design of the second study differed from the first in four respects: more subjects were run; they were required to make quantized heading estimates using two response modes, switch adjustment and numerical estimation; and the displays were designed to permit the use of shorter tracks, and to ensure a more equal sampling of heading direction over the 360-degree range than obtained in the first study.

Twenty men were used, 10 airmen and 10 civilians. In each subject group, three of the men were "experienced" subjects from the earlier heading estimation experiment.

The displays used consisted of bright vectors (straight lines) rear-projected on a dark screen, so as to measure $\frac{1}{32}$ inch in width, and in four lengths—1, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ inches. These vectors provided an unambiguous indication of heading even for very short display lengths. The tail (back end) of each vector was indicated by a dot, offset to one side so as not to represent a visual continuation of the vector itself.

Each subject viewed in random sequence 36 vectors for each of the four display lengths used, using each of the two response modes, making a total of

288 heading estimates. The particular headings chosen for display were selected so that each 10-degree interval of possible azimuth directions (001 to 010 degrees, 011 to 020, etc.) was represented by one vector of each of the four display lengths used.

In this second study, the rotary switch was not continuously adjustable. Instead, it was detented at 36 positions corresponding to the 10-degree azimuth intervals. Similarly, the subjects were not permitted to make numerical estimates guessing to the nearest degree. Instead, they had to report their estimates on push-button modules which provided only 10-degree accuracy. This equipment-constrained response quantizing meant that a subject could not estimate correctly in every case, even as a theoretical proposition. The best he could do was to minimize error size by making an optimal switch setting, or the nearest possible numerical estimate. For a vector displayed at an angle of 107 degrees, for example, an optimum estimate would be 110 degrees, a second best estimate would be 100 degrees, and so on.

TABLE 2

HEADING ESTIMATION ERRORS (IN DEGREES) FOR SECOND EXPERIMENT

Subjects	Length of displayed vector			
	1/8″	1/4″	1/2″	1″
Airmen				
Switch adjustment	9.9	7.7	9.1	10.5
Numerical estimation	14.6	11.7	10.9	13.1
Civilian				
Switch adjustment	6.2	5.7	5.0	5.3
Numerical estimation	9.6	6.7	8.3	7.7

Note.—Each entry is the mean error of 360 estimates, 36 made by each of 10 subjects, with 10-degree-interval response quantizing.

RESULTS AND DISCUSSION

The civilian group was again more accurate than the airmen. Both groups were more accurate estimating heading by switch adjustment than when they made numerical estimates reported by button pushing. They were also faster using switch adjustment. Both the number and kinds of estimation errors were consistently related to the particular heading direction displayed: fewer errors were made for headings approximating the direction of the four cardinal points (360°, 90°, 180°, and 270°); those errors that were made tended to emphasize the perceived discrepancy between the actual heading and these implicit vertical and horizontal references. Ambiguous differences were noted relating to the effect of displayed vector length on estimation accuracy.

The average estimation error for each subject group using the two different response modes is summarized in Table 2. Since these results represent all estimates made in this study, it is assumed that average error size is a legitimate comparative measure in spite of the enforced quantizing error for single responses.

Variance analysis along the same lines as described before, confirmed at the .001 level the differences noted in the first study: the civilians were more accurate as a group than the airmen ($F = 36.4$, $df = 1/18$) and switch adjustment was the more accurate response mode ($F = 36.9$, $df = 1/136$). It should be mentioned that this confirmation does not simply reflect the fact that some of the same subjects were used. Six men, it is true, did participate in both studies. However, the data from the 14 new subjects show the same differences, in kind and degree, between subject groups and between response modes. The chief point of interest is that the superiority of switch adjustment held up even when the constraints of response quantizing were made identical with those for numerical estimation.

The average estimation error for those subjects who participated in both studies increased by about a degree for both switch adjustment and numerical estimation, under the conditions of the second study. The difference is a small one, and we may conclude that 10-degree quantizing is almost if not quite equivalent to a continuous response capability in terms of expected average estimation error. This is confirmed by the plot of cumulative error distribution presented in Figure 2, which varies surprisingly little from the unconstrained response conditions of the first study.

In terms of response time for heading estimation, the switch adjustment mode which was slower in the first study turned out to be significantly faster in the second, averaging 4.1 seconds as compared with 5.5 seconds for numerical estimation. Presumably this reflects the fact that precise adjustments were not possible in the second study, and so the subjects could make a quicker, more casual se-

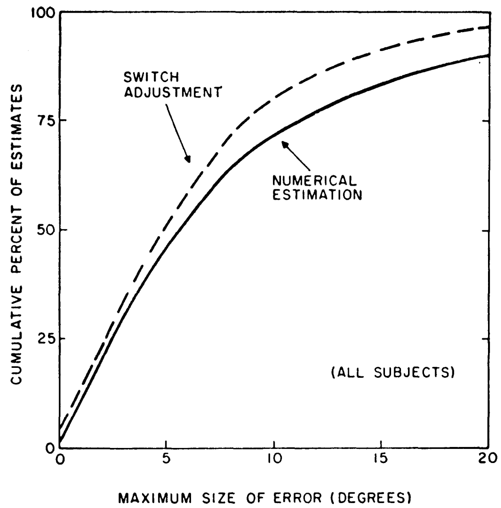

FIG. 2. Cumulative error distribution in second experiment.

lection among switch settings. Numerical estimates, on the other hand, had to be reported by button insertion rather than simply stated verbally, which would increase response time somewhat.

The picture with regard to the effect of displayed vector length is more confusing. The average estimation error, for all subjects using both response modes, is as follows:

Vector length (inches)	Average error (degrees)
$\frac{1}{8}$	10.1
$\frac{1}{4}$	8.0
$\frac{1}{2}$	8.3
1	9.2

Marginally significant effects on error attributable to display lengths were indicated by the variance analysis ($F = 3.97$, $df = 3/136$). Application of the range test to the differences in estimation accuracy associated with different displayed vector lengths resulted in a somewhat confusing conclusion: accuracy for $\frac{1}{8}$-inch vectors was less than for $\frac{1}{4}$-inch vectors (at the .01 level) and possibly less than for $\frac{1}{2}$-inch vectors (.05 level), but not significantly different than for 1-inch vectors. It is true, and somewhat reassuring that the shortest vectors, which provided the least adequate angular cues, resulted in the greatest average estimation error. However, it would be more encouraging if the accuracy differences between $\frac{1}{8}$-inch and still longer vectors than $\frac{1}{4}$-inch were also statistically reliable. This was not the case. In this connection, a possible alternative approach to the data, a chi square analysis based on observed frequency of optimal versus nonoptimal estimates, confirmed statistically reliable differences between subject groups, and between response modes, but none related to display length. The only clear conclusion from these present studies is that the adequate cue for angular perception is surprisingly short.

Data from the first study demonstrated a small error bias associated with right- versus left-handed switch adjustment. This seems to have been confirmed in the present context, in spite of the 10-degree detenting of the rotary switch. For those subjects making right-handed switch adjustments, 35% of their errors were clockwise, whereas subjects working left-handed made 52% clockwise errors. Chi square analysis confirms this difference ($\chi^2 = 85$, $p < .001$). There was *no* difference in error bias indicated by a corresponding analysis of right- and left-handed button insertion ($\chi^2 = .07$), which tends to rule out the alternative hypothesis that the particular subjects involved in this comparison had consistent perceptual biases.

A summary of errors as related to displayed direction is presented in Figure 3. Fewer errors were made in estimations of headings in directions approximating the cardinal azimuth points, 90, 180, 270, and 360 degrees, which represent implicit vertical and horizontal references. This effect seems to have been somewhat more pronounced for numerical estimation than for switch adjustment. In the case of switch adjustment, there seems to be both an improvement in accuracy at the cardinal points and for heading directions approximating quadrant bisections, near 45, 135, 225, and 315 degrees.

The directional bias of errors as related to displayed heading is presented in Figure 4. A trend seems to be apparent toward clockwise errors for headings somewhat clockwise of the cardinal directions, and counterclockwise errors for headings displaced somewhat in a counterclockwise direction from the horizontal or vertical. In brief, the subjects when they made errors seemed to emphasize the

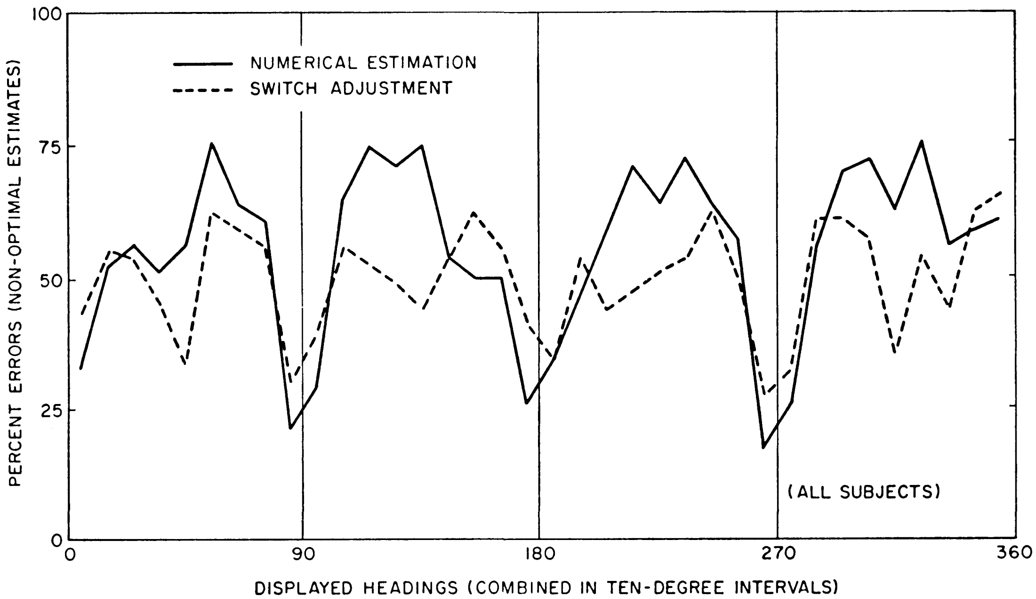

FIG. 3. Estimation errors as related to displayed heading in second experiment.

perceived disparity between a displayed heading approximating a horizontal or vertical direction and the implicit reference itself.

As it happens, the tendency toward response bias is greatest where the tendency toward error is least; i.e., for headings approximating vertical or horizontal directions as displayed. Thus, for practical purposes we might well be able to ignore such effects altogether. Moreover, these data, after all, are in some degree inferential. Because of the response quantizing feature of this study, we must rely on error frequency data for illustrating these differences. It is clear that an experimenter who is interested in an effective exploration of these phenomena should

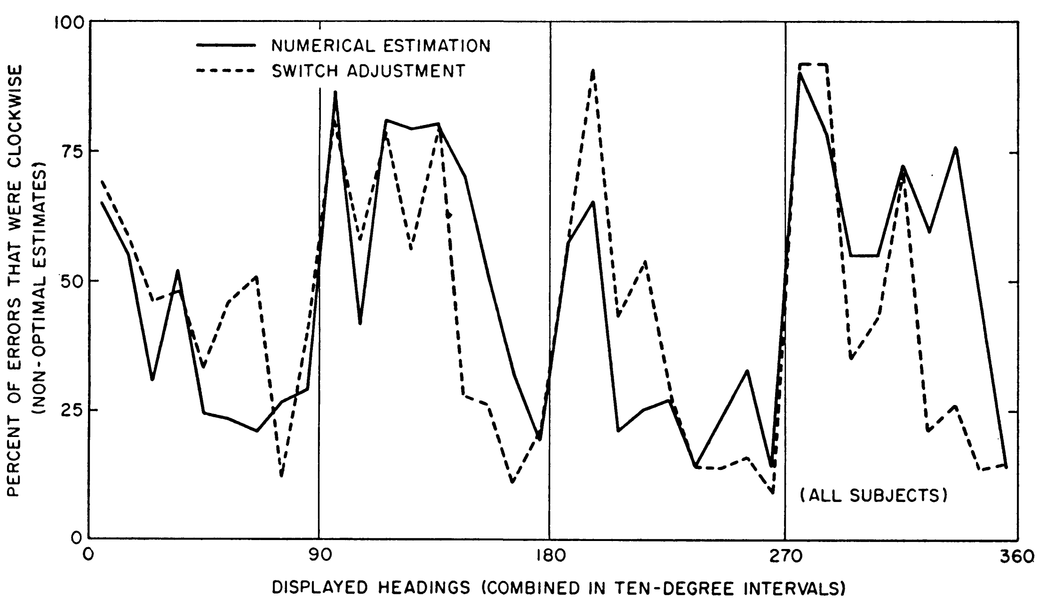

FIG. 4. Directional trend of estimation errors in second experiment.

permit his subjects to make continuous manual adjustments, and present a selection of displayed headings finely distributed over the 360-degree range. Since this does not describe the conditions of the present study, it was concluded that no statistical analysis of the present results was appropriate. However, the present results are suggestive.

Perhaps the most relevant previous work on this question is described in a series of research reports published by the Mount Holyoke College Psychophysical Research Unit (Kaufman, Reese, Volkmann, & Rogers, 1947; Reese, Volkmann, Rogers, & Kaufman, 1948; Rogers, Volkmann, Reese, & Kaufman, 1947). These dealt with estimation of bearings over the range from 350 degrees clockwise to 100 degrees. Because of differences in experimental procedure and their limited range of stimuli, it is difficult to draw comparisons with the present data. They did note a similar "anchoring" effect of error reduction in the vicinity of 360 and 90 degrees. However, there is no apparent confirmation in their data of the "sharpening" effect noted here, the exaggeration of perceived discrepancies from vertical and horizontal references.

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